First Fundamental Form or Metric :—

The quadratic eqn in du and dv of the form

$$ds^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2 = 0$$

[ds = Small arc length]

is Metric or First fundamental form.

And E, F and G is called the first fundamental Coefficient

where $E = r_1^2 = r_1 \cdot r_1$

$F = r_1 r_2 = r_2 \cdot r_1$

$G = r_2^2 = r_2 \cdot r_2$

$\boxed{H^2 = EG - F^2}$

here $r_1 = \frac{\partial r}{\partial u}$, $r_2 = \frac{\partial r}{\partial v}$

$r_{12} = \frac{\partial^2 r}{\partial u \partial v}$

$r_{21} = \frac{\partial^2 r}{\partial v \partial u}$

$A_n = \sum_{K=1}^{\infty} E(|f_{n_K} - f|) \geq \sigma_K$, $\quad B = \bigcap_{i=1}^{\infty} A_i$ —— (2)

We know that $\langle A_n \rangle$ is a monotonic decreasing sequence of measurable set

$\lim m(A_n) = m(B)$ —— (3)

If $E_1, E_2 \dots$ are measurable set and $E = \bigcap_{r=1}^{\infty} E_r$ and if $E_1 \supset E_2 \supset \dots$ then $m(E) = \lim_{n\to\infty} m(E_n)$

It follows from (1) and (2)

$m(A_n) < \sum_{K=n}^{\infty} \eta_K$ consequently

$\lim m(A_n) = 0$ —— (4)

as $\sum \eta_K$ is convergent. This $\Rightarrow m(B) = 0$, [by eqn (3)]

Let $y \in E - B$ be arbitrary. Then $y \notin B$

$y \notin B \Rightarrow y \notin A_{n_0}$ for some natural number n_0

$\Rightarrow y \notin E(|f_{n_K} - f| \geq \sigma_K) \quad \forall K \geq n_0$

$\Rightarrow |f_{n_K}(y) - f(y)| < \sigma_K \quad \forall K \geq n_0$

$\Rightarrow \lim_{K\to\infty} f_n(y) = f(y) \quad \forall y \in E - B$

Also $m(B) = 0$, Hence $\lim_{K\to\infty} f_{n_K} = f$ a.e on E.

Theorem (F. Riesz): Let $\langle f_n \rangle$ be a sequence of functions which converges in measure to the function f on a measurable set E. Then there exists a subsequence which also converges to the function f almost everywhere.

Proof:- Let $\langle \sigma_n \rangle$ be a monotonic decreasing sequence of Positive term st $\lim \sigma_n = 0$. Let $\sum \eta_K$ be a convergent series of (+)ve term st $\sum_{K=1}^{\infty} \eta_K = 0$

$\langle f_n \rangle$ converge in measure to f

$\Rightarrow \lim_{n\to\infty} m[E(|f_n - f| \geq \sigma_K)] = 0$

$\Rightarrow m[E(|f_n - f| \geq \sigma_K)] < \eta_K$ —— (1)

sequence $\langle \sigma_K \rangle$ we can construct a sequence $\langle n_K \rangle$

satisfying the condition (1), $\langle f_{n_K} \rangle$ is a subsequence of $\langle f_n \rangle$

Show that $\lim_{K\to\infty} f_{n_K} = f(x)$ a.e on E